Section 2-4, Mathematics 108

Average Rate of Change

Rate of Change of a linear function

First let's look at what rate of change means on a graph.

Suppose that the temperature in the Sahara desert from June 1st through August 15th is described the following function.

$T(d) = 104 + \frac{1}{4}d$ where d is days since June 1st.

That's our US anachronistic Fahrenheit scale for anyone who happens to come from a civilized country where they use the metric system.

So the temperature increases 1/4 of a degree each day.

We say the rate of change in temperature is .25 degrees per day.

Let's now say that the cost of a college education in the US is described by the function

$C(y) = \$16,000 + \$1000y$

Where y is the number of years since 2015.

The rate of change is \$1000 per year.

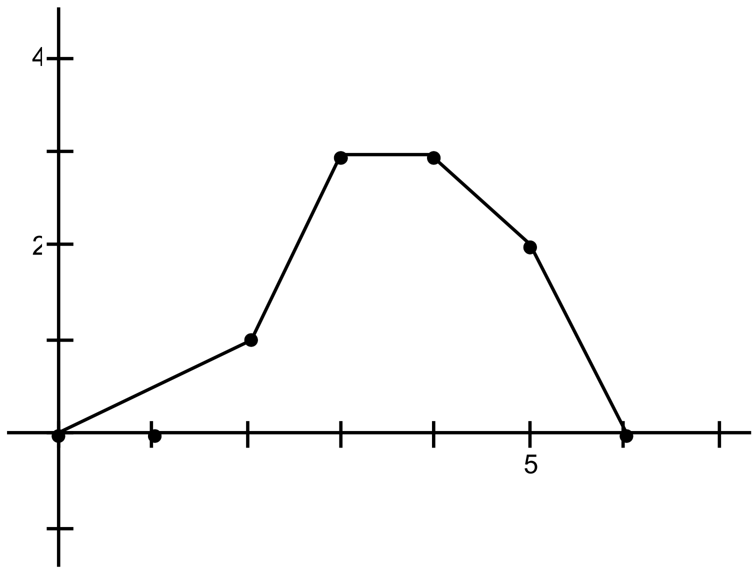
So the rate of change of a linear function is just a constant, the m in the slope intercept form

$y = mx + b$

Varying Rate of Change

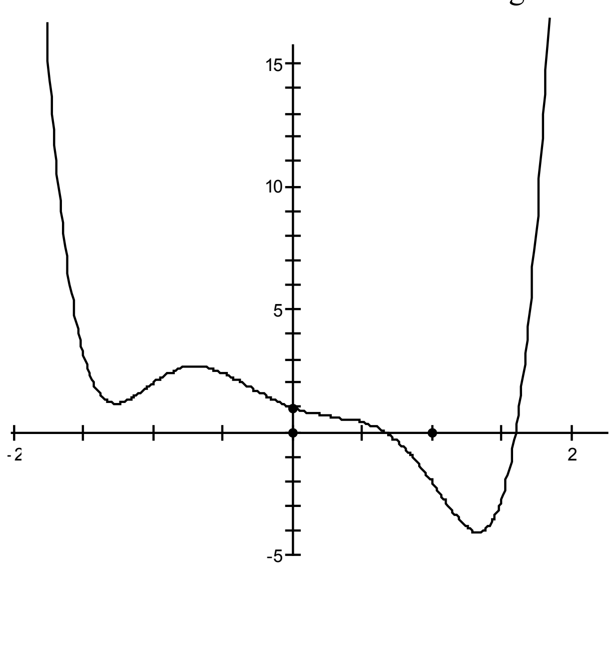
Many functions both mathematical and real world have a varying rate of change.

A really nice function with this property might look like this:



What's nice, but unrealistic about this is that there are five intervals in which the rate of change is constant, even though the rate will change from interval to interval.

Most mathematical functions will change continuously and smoothly, for exampe:



In Calculus you will learn how to deal with this type of function by assigning an absolute rate of change to each point on the graph, however we are left with a limitation right now. So instead we will find an approximation for the rate of change at a point by instead getting an average value for the rate of change over an interval.

We define the average rate of change of a function *f* on the interval [*a*,*b*] as

$$A_c(a,b) = \frac{f(b) - f(a)}{b - a}$$

Example:

What are the net change and average rate of change for the function

$f(x) = (x-3)^2$ on the interval [1,3]

The net change is just the difference in the value of the function between the end points of the interval so

$$NC = f(3) - f(1) = (3-3)^2 - (1-3)^2 = 0 - 4 = -4$$

Using our formula for the average rate of change

$$A_c(3,1) = \frac{f(3) - f(1)}{3-1} = \frac{(3-3)^2 - (1-3)^2}{3-1} = \frac{-4}{2} = -2$$